Functions and Composite Functions - Notes & Equations

Definition of Function

Let X and Y be sets.

A function from X to Y is a rule that assigns each element of X to exactly one element of Y.

Notation:
$f : X \to Y$

Important Notes:
1. X is called the Domain.
2. Y is called the Codomain.
3. f(x) is called the image of x.
4. The set of all images is called the Range.

$\text{Range}(f) = \{ y \in Y : y = f(x) \}$

Example 1

$f : R \to R$
$f(x) = x^2$

Domain = R
Codomain = R
Range = $[0, \infty)$

Example 2

$f : R \to R$
$f(x) = \sqrt{x}$

This function is not defined for negative numbers.

To make it well-defined:
Domain = $[0, \infty)$

Example 3

A = {1,2,3}

P(A) = { ∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, A }

Define:
f : P(A) → Z

f(B) = number of elements in B

Examples:
f(∅) = 0
f({2}) = 1
f({1,3}) = 2

Range(f) = {0,1,2,3}

Example 4

A = set of all 4-bit strings

f(s) = position of the most left zero in s

Examples:
f(1010) = 2
f(1100) = 3

But:
f(1111) = ?

Therefore, the function is NOT well-defined.

Composite Functions

Suppose:
f : A → B
g : B → C

Then:

$(g \circ f)(x) = g(f(x))$

Example 5

$f : A \to B$

$g : B \to C$

If:

f(1)=a

f(2)=c

f(3)=b

and

g(a)=2

g(b)=4

g(c)=1

Then:

$(g\circ f)(1)=4$

$(g\circ f)(2)=1$

$(g\circ f)(3)=4$

Example 6

$f(x)=3x+5$

$g(x)=x^2+1$

Find:

1) $(g\circ f)(2)$

f(2)=11

$g(11)=11^2+1=122$

Answer = 122

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2) $(g\circ f)(-3)$

$f(-3)=-4$
$g(-4)=(-4)^2+1=17$

Answer = 17

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3) $(g\circ f)(x)$

$(g\circ f)(x)=g(3x+5)$

$= (3x+5)^2 +1$

$= 9x^2 +30x +25 +1$

$= 9x^2 +30x +26$

Example 7

$f(x)=2x+1$
$g(x)=x^2$

Find:

$(g\circ f)(x)$

$= g(2x+1)$

$= (2x+1)^2$

$= 4x^2 +4x +1$

Now find:
$(f\circ g)(x)$

$= f(x^2)$

$= 2x^2 +1$

Therefore:

$(g \circ f)(x) \neq (f \circ g)(x)$