# <u>Discrete mathematics,</u>
# <u>HW#2- Sets - Functions</u>

1) **Determine whether each of these statements is true or false.**
   **a)** $0 \in \varnothing$ **b)** $\varnothing \in \{0\}$
   **c)** $\{0\} \subset \varnothing$ **d)** $\varnothing \subset \{0\}$
   **e)** $\{0\} \in \{0\}$ **f )** $\{0\} \subset \{0\}$
2) **What is the cardinality of each of these sets?**
   **a)** $\{a\}$ **b)** $\{\{a\}\}$
   **c)** $\{a, \{a\}\}$ **d)** $\{a, \{a\}, \{a, \{a\}\}\}$
3) **Let A = {a, b, c, d} and B = {y, z}. Find**
   **a)** $A \times B$. **b)** $B \times A$.
4) **How many different elements does A $\times$ B have if A has m elements and B has n elements?**
5) **Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}. Find**
   **a)** $A \cup B$. **b)** $A \cap B$.
   **c)** $A - B$. **e)** $A \oplus B$
6) **Find the sets A and B if A − B = {1, 5, 7, 8}, B − A ={2, 10}, and A $\cap$ B = {3, 6,9}.**
7) **Suppose that the universal set is U = {1, 2, 3, 4,5, 6, 7, 8, 9, 10}. Express each of these sets with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise.**
   **a)** $\{3, 4, 5\}$ **b)** $\{1, 3, 6, 10\}$
   **c)** $\{2, 3, 4, 7, 8, 9\}$
8) **Using the same universal set as in the last problem, find the set specified by each of these bit strings.**
   **a)** 11 1100 1111 **b)** 01 0111 1000
   **c)** 10 0000 0001
9) **Determine whether each of these functions from {a, b, c, d} to itself is one-to-one.**
   a. $f(a) = b, f(b) = a, f(c) = c, f(d) = d$
   b. $f(a) = b, f(b) = b, f(c) = d, f(d) = c$
   c. $f(a) = d, f(b) = b, f(c) = c, f(d) = d$
10) **Which functions in Exercise 10 are onto?**
11) **Determine whether each of these functions is a bijection from R to R.**
   **a)** $f(x) = -3x + 4$ **b)** $f(x) = -3x2 + 7$
12) **Let g(x)=$\lfloor x \rfloor$, find**
   **a)** $g^{-1}(\{0\})$. **b)** $g^{-1}(\{-1, 0, 1\})$.
   **c)** $g^{-1}(\{x \mid 0 < x < 1\})$.
13) **Conjecture a simple formula for an if the first 10 terms of the sequence {an} are 1, 7, 25, 79, 241, 727, 2185, 6559, 19681, 59047.**

**14) How can we produce the terms of a sequence if the first 10 terms are 1, 3, 4, 7, 11, 18, 29, 47, 76, 123?**

**15) Compound Interest Suppose that a person deposits \$10,000 in a savings account at a bank yielding 11% per year with interest compounded annually. How much will be in the account after 30 years?**

**16) Find the solution to each of these recurrence relations and initial conditions. Use an iterative approach.**

a. $a_n = 3a_{n-1}, a_0 = 2$

b. $a_n = a_{n-1} + 2, a_0 = 3$

c. $a_n = a_{n-1} + n, a_0 = 1$

**17) Suppose that the number of bacteria in a colony triples every hour. Set up a recurrence relation for the number of bacteria after n hours have elapsed. If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?**

**18) For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list.**

*a.* 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, . . .

*b.* 1, 2, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8, . . .

*c.* 3, 5, 8, 12, 17, 23, 30, 38, 47, . . .

*d.* 2, 16, 54, 128, 250, 432, 686, . . .

**19) Compute each of these double sums:**

**a)** $\sum_{i=1}^{2} \sum_{j=1}^{3} (i+j)$ **b)** $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i+3j)$

**c)** $\sum_{i=1}^{3} \sum_{j=0}^{2} i$ **d)** $\sum_{i=0}^{2} \sum_{j=1}^{3} ij$

**20) Let A =** $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$**, and B =** $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$**, Find**

**a) A $\vee$ B.** **b) A $\wedge$ B.** **c) A$\Theta$B.** **d) A$^{[3]}$**

**21) If A =** $\begin{bmatrix} 4 & -3 \\ 3 & -1 \\ 0 & -2 \\ -1 & 5 \end{bmatrix}$**, B =** $\begin{bmatrix} -1 & 3 & 2 & -2 \\ 0 & -1 & 4 & -3 \end{bmatrix}$ **and C=** $\begin{bmatrix} 1 & 0 & -2 & 3 \\ 2 & 1 & 4 & -1 \end{bmatrix}$**, Find:**

**a) A×B** **b) A$^T$**

**c) B×C** **d) A+5C**