Acknowledgement

• References and Textbooks

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2.Juan M. Morales , Antonio J. Conejo , Henrik Madsen, Pierre Pinson , Marco Zugno, “Integrating Renewables in Electricity Markets: Operational Problems”, 2013.

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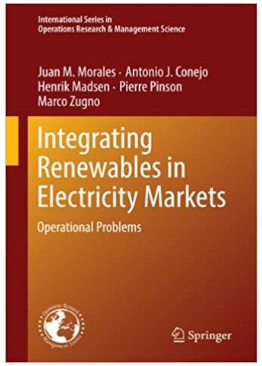

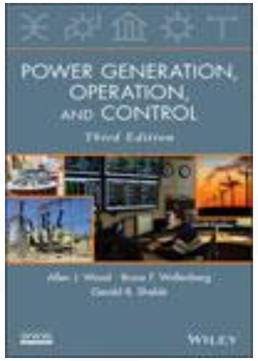

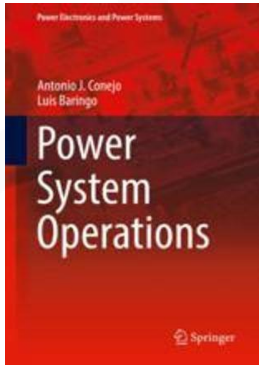

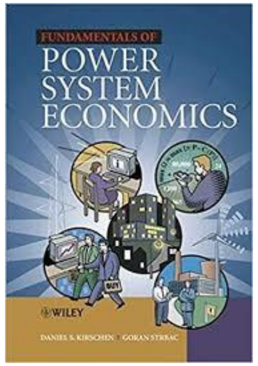

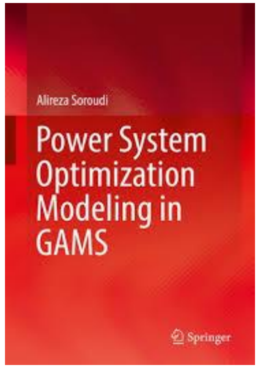

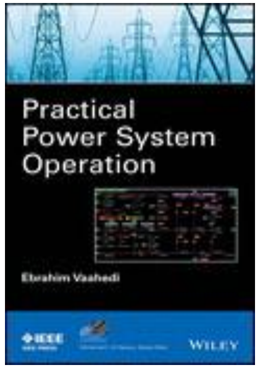

What

Column-and-Constraint Generation (CCG) algorithm

Robust set

$$\boldsymbol{u} \in \boldsymbol{\mathcal{U}}$$

ARO

$$
\min_{\boldsymbol{x}} \quad \text{s.t.} \quad \boldsymbol{h}(\boldsymbol{x}) = \mathbf{0}, \quad \boldsymbol{g}(\boldsymbol{x}) \leq \mathbf{0}
$$

$$
\max_{\boldsymbol{u}} \quad \text{s.t.} \quad \boldsymbol{u} \in \boldsymbol{\mathcal{U}}
$$

$$
\begin{aligned}
\min_{\boldsymbol{y}} \quad & \boldsymbol{c}_{\mathrm{I}}^{\top}\boldsymbol{x} + [\boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x},\boldsymbol{u})]^{\top}\boldsymbol{y} \\
\text{s.t.} \quad & \boldsymbol{A}(\boldsymbol{x},\boldsymbol{u}) \cdot \boldsymbol{y} = \boldsymbol{b}(\boldsymbol{x},\boldsymbol{u}) : \quad \boldsymbol{\lambda} \\
& \boldsymbol{D}(\boldsymbol{x},\boldsymbol{u}) \cdot \boldsymbol{y} \geq \boldsymbol{e}(\boldsymbol{x},\boldsymbol{u}) : \quad \boldsymbol{\mu}
\end{aligned}
$$

$\boldsymbol{c}_{\mathrm{I}}^\top \boldsymbol{x}$ is a constant

Third-level problem

$$
\begin{aligned}
\min_{\boldsymbol{y}} \quad & [\boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x}, \boldsymbol{u})]^\top \boldsymbol{y} \\
\text{s.t.} \quad & \boldsymbol{A}(\boldsymbol{x}, \boldsymbol{u}) \cdot \boldsymbol{y} = \boldsymbol{b}(\boldsymbol{x}, \boldsymbol{u}) : \quad \boldsymbol{\lambda} \\
& \boldsymbol{D}(\boldsymbol{x}, \boldsymbol{u}) \cdot \boldsymbol{y} \geq \boldsymbol{e}(\boldsymbol{x}, \boldsymbol{u}) : \quad \boldsymbol{\mu}
\end{aligned}
$$

Dual of the third-level problem

$$
\begin{array}{lll}
\min\limits_{\boldsymbol{y}} & [\boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x},\boldsymbol{u})]^\top \boldsymbol{y} & \\
\text{s.t.} & \boldsymbol{A}(\boldsymbol{x},\boldsymbol{u}) \cdot \boldsymbol{y} = \boldsymbol{b}(\boldsymbol{x},\boldsymbol{u}) : & \boldsymbol{\lambda} \\
& \boldsymbol{D}(\boldsymbol{x},\boldsymbol{u}) \cdot \boldsymbol{y} \geq \boldsymbol{e}(\boldsymbol{x},\boldsymbol{u}) : & \boldsymbol{\mu}
\end{array}
$$

$$
\begin{array}{ll}
\max\limits_{\boldsymbol{\lambda},\boldsymbol{\mu}} & [\boldsymbol{b}(\boldsymbol{x},\boldsymbol{u})]^\top \boldsymbol{\lambda} + [\boldsymbol{e}(\boldsymbol{x},\boldsymbol{u})]^\top \boldsymbol{\mu} \\
\text{s.t.} & [\boldsymbol{A}(\boldsymbol{x},\boldsymbol{u})]^\top \boldsymbol{\lambda} + [\boldsymbol{D}(\boldsymbol{x},\boldsymbol{u})]^\top \boldsymbol{\mu} = \boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x},\boldsymbol{u}) \\
& \boldsymbol{\lambda} : \quad \text{free} \\
& \boldsymbol{\mu} \geq \boldsymbol{0}
\end{array}
$$

Second-level problem and dual of the third-level problem merged

$$
\begin{aligned}
\max_{\boldsymbol{u},\boldsymbol{\lambda},\boldsymbol{\mu}} \quad & [\boldsymbol{b}(\boldsymbol{x},\boldsymbol{u})]^\top \boldsymbol{\lambda} + [\boldsymbol{e}(\boldsymbol{x},\boldsymbol{u})]^\top \boldsymbol{\mu} \\
\text{s.t.} \quad & \boldsymbol{u} \in \boldsymbol{\mathcal{U}} \\
& [\boldsymbol{A}(\boldsymbol{x},\boldsymbol{u})]^\top \boldsymbol{\lambda} + [\boldsymbol{D}(\boldsymbol{x},\boldsymbol{u})]^\top \boldsymbol{\mu} = \boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x},\boldsymbol{u}) \\
& \boldsymbol{\lambda} : \quad \text{free} \\
& \boldsymbol{\mu} \geq \boldsymbol{0}
\end{aligned}
$$

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Subproblem: $\boldsymbol{x} = \boldsymbol{x}^{(\boldsymbol{\nu}-\mathbf{1})}$ fixed

Subproblem: $\boldsymbol{x} = \boldsymbol{x}^{(\nu-1)}$ fixed

$$
\begin{array}{ll}
\max\limits_{\boldsymbol{u},\boldsymbol{\lambda},\boldsymbol{\mu}} & [\boldsymbol{b}(\boldsymbol{x}^{(\nu-1)},\boldsymbol{u})]^\top \boldsymbol{\lambda} + [\boldsymbol{e}(\boldsymbol{x}^{(\nu-1)},\boldsymbol{u})]^\top \boldsymbol{\mu} \\
\text{s.t.} & \boldsymbol{u} \in \boldsymbol{\mathcal{U}} \\
 & [\boldsymbol{A}(\boldsymbol{x}^{(\nu-1)},\boldsymbol{u})]^\top \boldsymbol{\lambda} + [\boldsymbol{D}(\boldsymbol{x}^{(\nu-1)},\boldsymbol{u})]^\top \boldsymbol{\mu} = \boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x}^{(\nu-1)},\boldsymbol{u}) \\
 & \boldsymbol{\lambda} : \quad \text{free} \\
 & \boldsymbol{\mu} \geq \boldsymbol{0}
\end{array}
$$

$$\Downarrow$$

$$\boldsymbol{u}^{(\nu)}, \boldsymbol{\lambda}^{(\nu)}, \boldsymbol{\mu}^{(\nu)}$$

Upper bound: $\boldsymbol{x}$ arbitrarily fixed

($\boldsymbol{x}$ to be fixed to minimize the objective function)

$$\mathrm{UB}_\nu = \boxed{\boldsymbol{c}_{\mathbf{I}}^\top \boldsymbol{x}^{(\nu-1)}} + \boxed{[\boldsymbol{b}(\boldsymbol{x}^{(\nu-1)}, \boldsymbol{u}^{(\nu)})]^\top \boldsymbol{\lambda}^{(\nu)} + [\boldsymbol{e}(\boldsymbol{x}^{(\nu-1)}, \boldsymbol{u}^{(\nu)})]^\top \boldsymbol{\mu}^{(\nu)}}$$

Master problem: $\boldsymbol{u} = \boldsymbol{u}^{(k)}, \boldsymbol{k} = \mathbf{1}, \ldots, \boldsymbol{\nu},$ fixed

Master problem: $\boldsymbol{u} = \boldsymbol{u}^{(k)}, \boldsymbol{k} = \mathbf{1}, \ldots, \boldsymbol{\nu},$ fixed

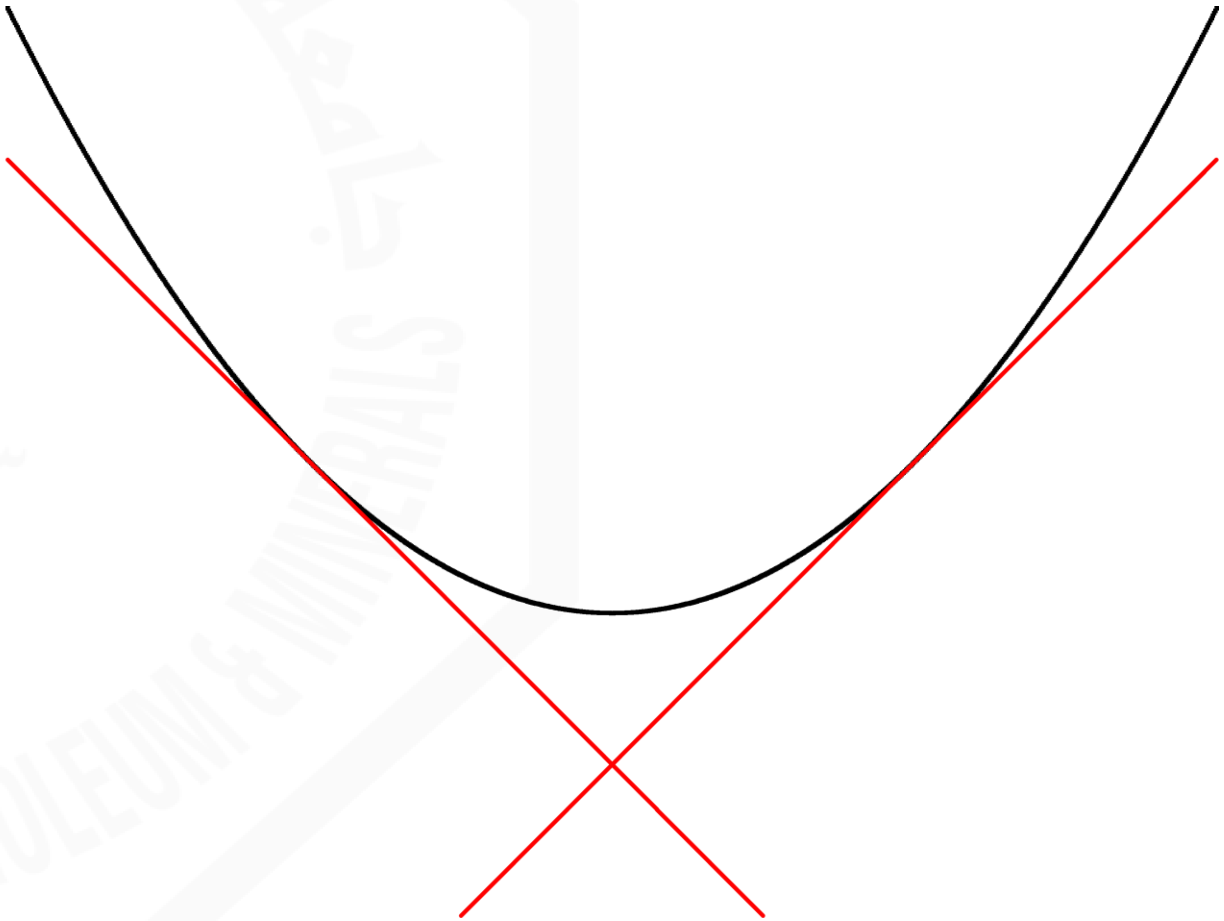
$$
\begin{array}{lll}
\min\limits_{\boldsymbol{x},\boldsymbol{\eta},\boldsymbol{y}^{(k)},\boldsymbol{k}=1,\ldots,\boldsymbol{\nu}} & \boldsymbol{c}_{\mathbf{I}}^{\top}\boldsymbol{x} + \boldsymbol{\eta} & \\
\text{s.t.} & \boldsymbol{h}(\boldsymbol{x}) = \mathbf{0} & \\
 & \boldsymbol{g}(\boldsymbol{x}) \leq \mathbf{0} & \\
 & \boldsymbol{\eta} \geq [\boldsymbol{c}_{\mathbf{O}}(\boldsymbol{x}, \boldsymbol{u}^{(k)})]^{\top}\boldsymbol{y}^{(k)} & \boldsymbol{k} = \mathbf{1}, \ldots, \boldsymbol{\nu} \\
 & \boldsymbol{A}(\boldsymbol{x}, \boldsymbol{u}^{(k)}) \cdot \boldsymbol{y}^{(k)} = \boldsymbol{b}(\boldsymbol{x}, \boldsymbol{u}^{(k)}) & \boldsymbol{k} = \mathbf{1}, \ldots, \boldsymbol{\nu} \\
 & \boldsymbol{D}(\boldsymbol{x}, \boldsymbol{u}^{(k)}) \cdot \boldsymbol{y}^{(k)} \geq \boldsymbol{e}(\boldsymbol{x}, \boldsymbol{u}^{(k)}) & \boldsymbol{k} = \mathbf{1}, \ldots, \boldsymbol{\nu}
\end{array}
$$

$$\Downarrow$$

$$\boldsymbol{x}^{(\nu)}, \boldsymbol{\eta}^{(\nu)}$$

$$(\& \quad \boldsymbol{y}^{(k)}, \boldsymbol{k} = \mathbf{1}, \ldots, \boldsymbol{\nu})$$

Lower bound: bounded from below $(\boldsymbol{\eta} \geq [\boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x}, \boldsymbol{u}^{(k)})]^\top \boldsymbol{y}^{(k)})$



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Lower bound: bounded from below ($\boldsymbol{\eta} \geq [\boldsymbol{c}_{\mathrm{O}}(\boldsymbol{x}, \boldsymbol{u}^{(k)})]^\top \boldsymbol{y}^{(k)}$)

$$\mathrm{LB}_\nu = \boldsymbol{c}_{\mathrm{I}}^\top \boldsymbol{x}^{(\nu)} + \boldsymbol{\eta}^{(\nu)}$$

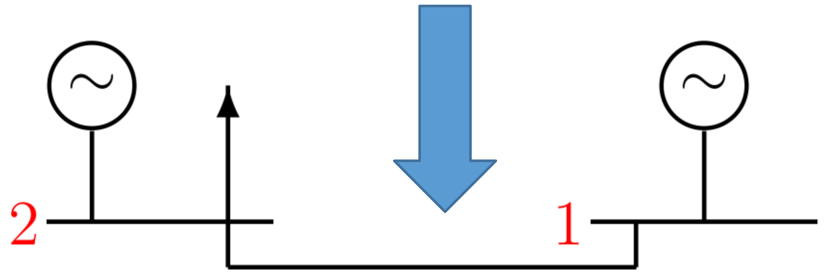
1. Solve the SP and get $\boldsymbol{u}^{(\nu)}$ (with previous or initial $\boldsymbol{x}^{(\nu-1)}$)
2. Compute the upper bound UB_ν
3. Solve the MP and get $\boldsymbol{x}^{(\nu)}$
4. Compute the lower bound LB_ν
5. If $\mathrm{UB}_\nu - \mathrm{LB}_\nu$ is small enough, stop; otherwise continue in 1

Zeng, B., and Zhao, L. "Solving two-stage robust optimization problems using a column-and-constraint generation method." Operations Research Letters, 41, 5 (2013), 457-461.

Bertsimas, D., Litvinov, E., Sun, X. A., Zhao, J. and Zheng, T. "Adaptive robust optimization for the security constrained unit commitment problem." IEEE Transactions on Power Systems, 28, 1 (2013), 52-63.

Simple adaptive robust optimization example

By utilizing GCC approch, implement the following Adaptive Robust Optimization using GAMS



A transmission expansion planner has the option of building just one of two alternatives transmission lines of capacity either 2 or 4 to connect electrical nodes 1 and 2.

The per unit building cost for the transmission lines is 1.

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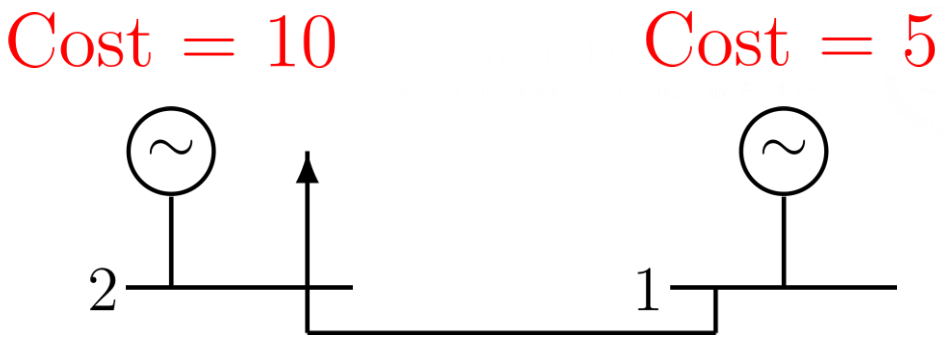

At node 1, a generator of unlimited capacity and per unit production cost of 5 is available, while in node 2 another generator of unlimited capacity and per unit production cost of 10 is available.

The demand is located at node 2 and can take solely two values, either 3 or 5.

ARO

$$
\begin{aligned}
\min_{x\in\{2,4\}} \quad \max_{d\in\{3,5\}} \quad \min_{y_1,y_2\geq 0} \quad & z = 1x + 5y_1 + 10y_2 \\
\text{s.t.} \quad & y_1 + y_2 = d \\
& y_1 \leq x
\end{aligned}
$$

where x represents the capacity of the transmission line to be built,

d the unknown demand, and

y_1 and y_2, the production of the units at nodes 1 and 2, respectively.

ARO

$$\min_{\mathbf{x}\in\mathcal{X}} \quad \max_{\mathbf{u}\in\mathcal{U}} \quad \min_{\mathbf{y}\in\mathcal{Y}(\mathbf{x},\mathbf{u})} \quad f(\mathbf{x},\mathbf{y},\mathbf{u})$$

On the computational side,

we note that tri-level problems require a specific solution algorithm that generally relies on

merging the second- and third-level problems using duality theory and

decomposing the resulting bi-level problem through a column-and-constraint generation algorithm.

Thank you!

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